All E Maths Formulas for O levels E Maths by Ethan Wu

Chapter 1: Indices

$a^5 = a \times a \times a \times a \times a$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{m \times n}$

$(ab)^n = a^n b^n$

$$\left(\frac{\mathbf{a}}{\mathbf{b}}\right)^{\mathbf{n}} = \frac{\mathbf{a^n}}{\mathbf{b^n}}$$

$a^0 = 1$

$a^{-n} = \frac{\mathbf{1}}{\boldsymbol{a^n}}$

$\left(\frac{3}{4}\right)^{-2} = \left(\frac{\mathbf{4}}{\mathbf{3}}\right)^2$

$a^{m/n} = (\sqrt[n]{a})^m$

Chapter 2: More about Quadratic Equations

4 methods of solving quadratic equations

Factorization

$ax^2 + bx + c = 0$

By factorisation,

$(px + q)(rx + s) = 0$

$X = -\frac{q}{p}$ or $x = -\frac{s}{r}$

General solution

The roots of $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the square method

$x^2 + bx + c = 0$

By completing the square

$(x + \frac{b}{2})^2 = q$

$x + \frac{b}{2} = \pm\sqrt{q}$

$x = -\frac{b}{2} \pm \sqrt{q}$

Graphical method

By drawing the graph and finding the x coordinates where the graph cuts the x axis

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a+b)(a-b)$

Discriminant = $b^2 - 4ac$

$b^2 - 4ac > 0$ real and distinct roots

$b^2 - 4ac = 0$ real and repeated roots

$b^2 - 4ac < 0$ roots are not real

Chapter 3: Linear Inequalities

1. Add or subtract numbers from each side of the inequality

e.g. 10 - 3 < x - 3

2. Multiply or divide numbers from each side of the inequality by a constant

e.g. $\frac{10}{3} < \frac{x}{3}$

3. Multiply or divide by a negative number AND REVERSE THE INEQUALITY SIGNS

e.g. 10 < x becomes $\frac{10}{-3} > \frac{x}{-3}$

4. Multiply or divide by an unknown= CANNOT BE DONE!

Chapter 4: Conditions of Congruence and Similarity

Prove congruency (same shape, same size)

SSS

SAS

ASA

RHS (the hypotenuse and one side of the triangle must be equal to the corresponding side)

Prove similar triangle (same shape, different size)

AAA

SSS (side-side-side)

SAS

Ratio of length of similar triangles

Ratio of area

$\frac{a_1}{a_2} = (\frac{x_1}{x_2})^2$

Ratio of volume of similar triangles

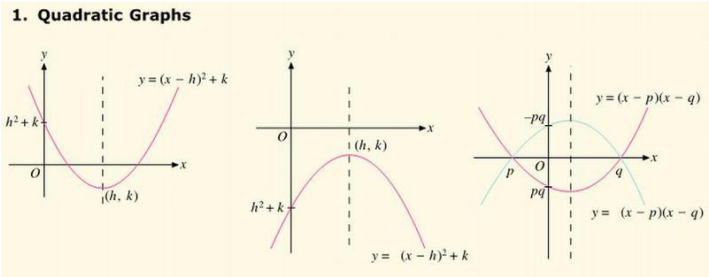
$\frac{v_1}{v_2} = (\frac{x_1}{x_2})^3$

Ratio of area of triangles with same height (b represent base)

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$

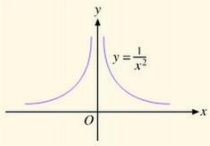
Chapter 5: Functions and Graphs

Graph of $y=ax^n$, where n=0, n=1, n=2, n=3,-2,-1

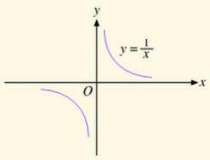


2. Graphs of Power Functions $y = ax^n$

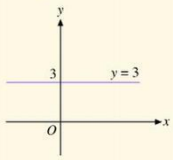
(a) $n = -2$, $a = 1$



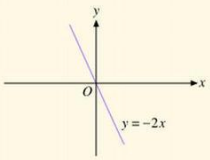
(b) $n = -1$, $a = 1$



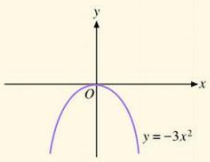
(c) $n = 0$, $a = 3$



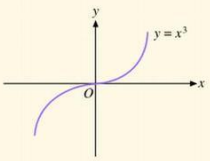
(d) $n = 1$, $a = -2$



(e) $n = 2$, $a = -3$



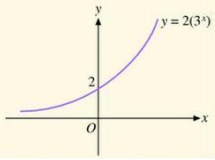
(f) $n = 3$, $a = 1$



3. Graphs of Exponential Functions $y = ka^x$

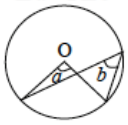
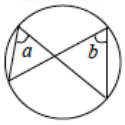
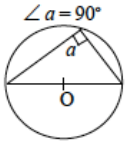
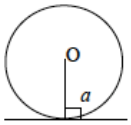
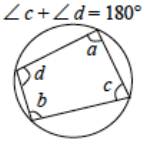
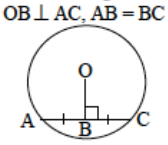
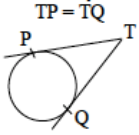
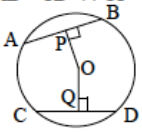
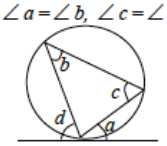
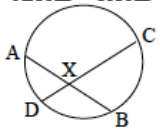
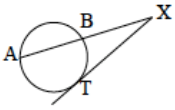
When a is a positive integer greater than 1 and $k > 0$,

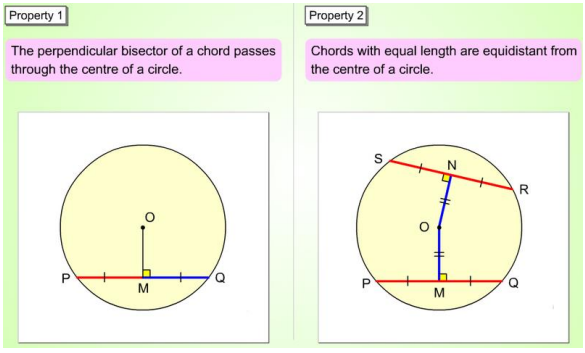
- $y > 0$ for all x,
- the graph passes through the point $(0, k)$,
- the graph increases rapidly for large values of x,
- the graph becomes close to the x-axis when x tends to the left end of the x-axis.



Chapter 6: Properties of Circles

Interior ∠s
Vert opp ∠s
∠s in alt segment
Right angle in semicircle
∠s in same segment (Alternate segment theorem)
∠ at centre = 2∠ at circumference
∠sum of Δ
Base ∠s of isos Δ
∠s at a point
Adj ∠s on a straight line
∠s in opp segments
Ext ∠ of a cyclic quad
Corr ∠s, Ab//CD
Tangent perpendicular to radius
Tangents from ext point
Alt ∠s, AB//CD

∠ at Centre $\angle a = 2\angle b$ 	**∠s in Same Segment** $\angle a = \angle b$ 	**∠ in Semi-Circle** $\angle a = 90°$ 	**Radius ⊥ Tangent** $\angle a = 90°$ 
Opp. ∠s of Cyclic Quadrilateral $\angle a + \angle b = 180°$ $\angle c + \angle d = 180°$ 	**⊥ bisector of chord passes through centre** OB ⊥ AC, AB = BC 	**Tangents from external point** TP = TQ 	**Equal chords equidistant from centre** AB = CD ↔ OP = OQ 
Alternate Segment Theorem $\angle a = \angle b,\ \angle c = \angle d$ 	**Intersecting Chords Theorem** AX·XB = CX·XD 	**Tangent-Secant Theorem** $AX \cdot BX = TX^2$ 	



Chapter 7: Trigonometry

$$\sin\theta = \frac{\text{opp}}{hyp}$$

$$\cos\theta = \frac{\text{adj}}{hyp}$$

$$\tan\theta = \frac{\text{opp}}{adj}$$

Sin rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cos rule

$c^2 = a^2 + b^2 - 2abcosC$

Area rule

Area = ½ absinC

Chapter 8: Applications of Trigonometry

Bearings

Angle of elevation = angle of depression same value

Problems in 3 dimensions

Chapter 9: Coordinate Geometry

Coordinates (x,y)

Length of line = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

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Gradient of line = $\frac{y_1 - y_2}{x_1 - x_2}$

Equation of line y = mx + c

Chapter 10: Arc Length and Sector Areas

S (arc length)=r (radius) x θ(angle in radian)

Area = ½rs or ½r^2θ

π rad = 180°

1 rad = 180°/π

1° = π/180 rad

Chapter 11: Quartiles and Percentiles

Range quartile = largest value – smallest value

Interquartile range = upper quartile (75^{th}) – lower quartile (25^{th})

Chapter 12: Standard Deviation

$$SD = \sqrt{\frac{\sum(x - \bar{x})^2}{N}}$$

$$\text{Or} = \sqrt{\frac{\sum x^2}{N} - (\frac{\sum x}{N})^2}$$

$$\text{Grouped Data} = \sqrt{\frac{\sum fx^2}{\sum f} - (\frac{\sum fx}{f})^2}$$

Chapter 13: Probability

Mutual exclusive, independent

If 2 events A and B are independent of each other, then the probability of both A and B occurring is found by P(A) x P(B)

Mutually exclusive

If it is impossible for both events A and B to occur, then the probability of A or B occurring is P(A) and P(B)

Chapter 14: Matrices ***

Addition and subtraction of matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & r \\ q & s \end{pmatrix} = \begin{pmatrix} a+p & b+r \\ c+q & d+s \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} - \begin{pmatrix} p & r \\ q & s \\ t & u \end{pmatrix} = \begin{pmatrix} a-p & b-r \\ c-q & d-s \\ e-t & f-u \end{pmatrix}$$

Scalar multiplication of matrix

$$k\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} ka & kb & kc \\ kd & ke & kf \end{pmatrix}$$

Multiplication of matrix

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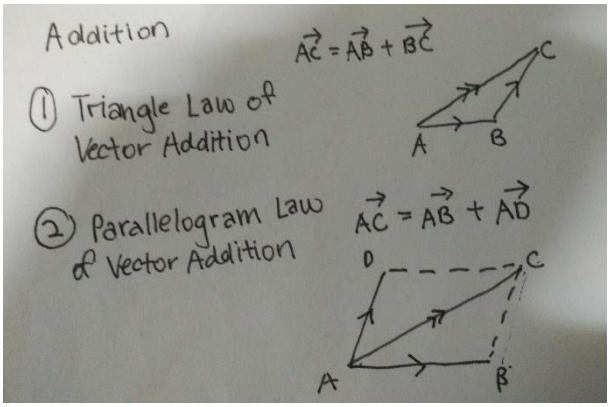
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} p & r \\ q & s \end{pmatrix} = \begin{pmatrix} ap+bq & ar+bs \\ cp+dq & cr+ds \end{pmatrix}$$

Chapter 15: Vectors in Two Dimensions ***

Vector expressed as either $\overrightarrow{PQ}$ or a

Triangle law of vector addition
Parallelogram law of vector addition
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BD}$



u,v,w are vectors
u + v = v + u (commutative law)
(u+v)+w = u+(v+w)
u-v = u+(-v)
a + (-a) = 0 (for any vector)

a and -a

Scalar multiplication
u,v vectors m,n numbers
m(nu) = n(mu) = (mn)u
(m+n)u = mu+nu
m(u+v) = mu+mv

position vector
O is origin
$\overrightarrow{OP}$ is position vector of P with respect to reference point 0

Vectors on coordinate plane
$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ = column vector, from O to P, move 2 units right in x direction and 3 units up in y direction
Magnitude = $\sqrt{x^2 + y^2}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$
$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1+x_2 \\ y_1+y_2 \end{pmatrix}$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ y_1 - y_2 \end{pmatrix}$$
$$k\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

Chapter 16: Set Language and Notation

$\in$: Element of

$\notin$: Not a element of

$n(a)$: The **number** of elements in a Set

$\subset$: A proper subset

$\subseteq$: A subset

$\varnothing$ or $\{\}$: Empty Set

ε : Universal Set

A': Complement of a Set

$\cup$: Union of 2 Sets (dont repeat element)

$\cap$: Intersection of sets

Chapter 17: Mathematics in Practical Situations

Profit/loss = selling price – cost price

% profit and loss = $\frac{net\ profit/loss}{cost\ price} \times 100\%$

Simple interest and compound interest

Simple interest I = P x i% x n

P is principal

i is interest rate

n is no of years

Compound interest A = $P(1 + \frac{i}{100})^n$

Hire purchase- payment for commodity is made in instalments over a period of time

Interest given on a flat rate basis (simple interest)

Utility bill = usage x rate

Money exchange

If S$1 = $m,

Then $1 = S$1/m

Taxation is calculated using a progressive tax rate on chargeable income

Chargeable income = assessable income – personal relief

Assessable income = annual income - donation

Property tax = annual value x tax rate

Property tax rate for owner-occupied properties 4%

Property tax rate for other properties 10%

Chapter 18: Graphs in Practical Situations

Distance-time graph, speed-time graph

Speed = gradient of distance-time graph

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Distance travelled = area under speed-time graph
Acceleration = change in speed/time = gradient of speed-time graph

Additional Formula Required

Mensuration

Cone

$SA = \pi rs + \pi r^2$

$V = 1/3\pi r^2 h$

Cylinder

$SA = 2\pi rh + 2\pi r^2$

$V = \pi r^2 h$

Sphere

$SA = 4\pi r^2$

$V = 4/3\pi r^3$

Pyramid

V= 1/3 x base area x h

interior angle $= \frac{(n-2)x180}{n}$

exterior angle $= \frac{360}{n}$

Angles

acute <90°

obtuse 90° < a <180°

reflex 180° < a < 360°